On the Structure of Mechanical Models illustrating some Properties of the Æther

1885 Proc. Phys. Soc. London 7 74

(http://iopscience.iop.org/1478-7814/7/1/310)

equation in terms of v to solve as the result of equating the differential to zero, which would, by the introduction of the proper constants, give the value of electromotive force at which the total cost becomes a minimum. In their paper, Professors Ayrton and Perry have calculated to a fraction of a volt what this economical potential is. As, however, the characteristic equations are only approximate, it seems hardly necessary to do more than obtain a similar approximate expression for the economical working.

At the Edison lamp-factory in America calculations were made, on the assumption of a particular type of lamp and length of life and cost of power, to ascertain the ratio of lampage to total cost, which made the total cost a minimum, and the result appeared to be to fix it at about 16 per cent. These calculations are therefore in singular accord with the deduction of theory based on determination of the constants of the characteristic curves, arrived at both by graphic and analytical methods.

X. *On the Structure of Mechanical Models illustrating some Properties of the Æther. By* Prof. GEORGE FRANCIS FITZGERALD, *F.R.S.**

THE elements of which the model is constructed consist of pairs of wheels so geared together that when one of them rotates it causes the second to rotate in the same direction. The simplest way of effecting this is to connect them by a band, and this is sufficient for a one-dimensional model. Such a model may be constructed by fixing a number of wheels with their axes parallel and at right angles to a plane, and connecting each wheel with its neighbours by elastic bands. This represents a nonconducting region of the æther. A perfectly conducting region is one in which there are no bands, and a partially conducting region would be represented by the bands slipping more or less. A short description of how electrostatic, electrokinetic, and luminiferous phenomena are illustrated by such a one-dimensional model will be clearer than the corre-

* Read March 28, 1885.

sponding description of the tridimensional model, the structure of which is the special purpose of this paper.

As an illustration of an electrostatic phenomenon, consider two conducting regions separated by a nonconducting region everywhere except along one line where the bands are removed. If anywhere in this line a rotation in opposite directions be communicated to the wheels that abut on it, then all the wheels in the nonconducting region will be turned more or less. If anywhere two neighbouring wheels turn equally, there is no straining of the band connecting them; but if one turn more than the other, the connecting band is strained and one side becomes tight and the other loose. Now it will be found in the case considered that all the bands are strained, and that all the tight sides are turned towards one conductor and all the loose sides towards the other. This represents the charging of the two conductors in opposite ways. The strain of the bands in any element of the medium represents the polarization of the element, and the line joining the tight and loose sides is the direction of polarization or of electric displacement. The energy of the system is in the form of this straining of the bands, which produces stresses tending to restore the unstrained condition. With a given strain at their respective surfaces, there would be more elements involved and more energy in the medium when the conductors are far apart than when near, showing that if we could represent in any way the fact that conductors can move through the æther there would be forces tending to produce this motion, or, in other words, there would be attraction between these oppositely electrified bodies. As, however, the model does not illustrate the connection between matter and æther, neither this nor magnetic attractions are represented, nor have electromotive forces such as exist in cells been represented. If the forces that have been supposed to turn the wheels along the conducting line connecting the two conducting regions cease to act, the state of strain will disappear, and what represents an electric discharge along this line will take place. All along this line, during the discharge, the wheels at opposite sides will be rotating in opposite directions; so that this is what represents an electric current at any point. It is the same as an electric displacement; and in a nonconductor such opposite rotation is resisted by the stresses in the band, but in a conductor it may take place to any extent. During the discharge the whole of the nonconducting region is full of rotating wheels, and their axes of rotation are at right angles to the direction of discharge. Their velocity of rotation evidently represents the magnetic force accompanying the discharge, and the momentum of the wheels represents the electrokinetic momentum of the current, *i.e.* its self-induction. This is further illustrated by this, that if the frictional resistance be small enough, this momentum will carry the wheels beyond their positions of equilibrium, and there will result an oscillating discharge, such as occurs when an electric condenser is discharged through a sufficiently small resistance. If we suppose a certain amount of frictional resistance at any point along the line of discharge, we may see that the energy expended on friction is conveyed to the place by the bands in the surrounding nonconductor and comes in at the side of the conductor, in accordance with Prof. Poynting's theorem as to the direction of the flow of energy in an electrodynamic system.

The mutual induction of two circuits may also be illustrated by the model. Sufficient has been explained, however, to show how electrostatic and electrokinetic phenomena are represented on the model. If a sudden movement of rotation be communicated to any set of wheels, it is evident that inertia will prevent their neighbours being instantaneously turned, while the connecting bands will be strained. Rotation will, however, be communicated to the neighbouring wheels, and from them to their neighbours, by a process which is a species of wave-propagation. If we consider the nature of the disturbance which is thus propagated, we see that it consists in a rotation whose axis is at right angles to the direction of propagation, and of a polarization of the bands which is at right angles both to the axis of rotation and to the direction of wave-propagation. These are respectively a magnetic and an electric displacement, which are at right angles to one another and to the direction of wave-propagation. This is exactly in accordance with Maxwell's electromagnetic theory of light-propagation. It is thus seen how the same model that can represent electrostatic and electromagnetic phenomena also illustrates luminiferous phenomena by its small oscillations.

If we try to produce a tridimensional model by means of wheels geared together by bands, we are met by the following difficulty. The energy of the model we have been considering may be represented in the following way:—Let ζ represent the angular rotation of any wheel from a given position; then the kinetic energy of an element will evidently be proportional to $\dot{\zeta}^2$, while the potential energy of an element, depending as it does on the difference of rotation of neighbouring wheels, will be proportional to $\left(\frac{d\zeta}{dx}\right)^2 + \left(\frac{d\zeta}{dy}\right)^2$. If we were to build up a tridimensional model by simply putting together three such systems of wheels in three rectangular planes, the kinetic energy of an element of the model would be proportional to

$$\dot{\xi}^2 + \dot{\eta}^2 + \dot{\zeta}^2,$$

but its potential energy would be simply

$$\overline{\frac{d\zeta}{dx}}\Big|^2 + \overline{\frac{d\zeta}{dy}}\Big|^2 + \overline{\frac{d\eta}{dz}}\Big|^2 + \overline{\frac{d\eta}{dx}}\Big|^2 + \overline{\frac{d\xi}{dy}}\Big|^2 + \overline{\frac{d\xi}{dz}}\Big|^2,$$

instead of

$$\left(\frac{d\zeta}{dy} - \frac{d\eta}{dz}\right)^2 + \left(\frac{d\xi}{dz} - \frac{d\zeta}{dx}\right)^2 + \left(\frac{d\eta}{dx} - \frac{d\xi}{dy}\right)^2,$$

which is what it should be if it is to represent the æther.

The electrostatic and electrokinetic energies of an element of the æther may be expressed in this form by assuming ξ, η, and ζ such that their velocities $\dot{\xi}$, $\dot{\eta}$, and $\dot{\zeta}$ are α, β, and γ, the components of the magnetic force at the element. Now simple band-gearing will not enable us to arrange that no straining shall result, when, for instance, $\frac{d\eta}{dx} = \frac{d\xi}{dy}$, but this may be accomplished by the following arrangement:—Each element of the æther is to be represented by a cube, on each edge of which there is a paddle-wheel. Thus on any face of the cube there will be four paddle-wheels. Now, if any opposite pair of paddle-wheels on a face rotate by different amounts, they will tend to pump any liquid in which the whole element is immersed into or out of the cube, and if the sides of the cube be elastic there will be a stress which will tend to stop this differential rotation of the wheels. If, however, the other pair also rotate by different amounts they may undo what the first pair do; and thus the stress will depend on the difference between the differential rotations of these opposite pairs of wheels, *i.e.* on $\frac{d\eta}{dx}-\frac{d\xi}{dy}$.

In order that these four wheels may not similarly work with any other wheel in the cube it is necessary to place diaphragms, cutting the cube into six cells, each a pyramid standing on a face of the cube. These must be so made that liquid may not be able to pass from one cell to another through the diaphragm, nor beside the paddle-wheels. In order actually to effect this, the floats on the paddle-wheels would have to be drawn down while passing the diaphragm (of course these mechanical details could hardly be carried out so as to work with sufficiently little friction for the working of any actual model to approximate sufficiently closely to that of the æther for it to be worth while attempting to construct it). The faces of the cube should be filled up with diaphragms past which the paddle-wheels should pump liquid, and whose elasticity should be the means of storing electrostatic energy in the medium. It may be worth while pointing out some of the results of having shown that the energy of a disturbance of this medium would be represented by the same equations as those Maxwell has shown to hold for the æther. The most complicated results follow from supposing the faces of the cubes, of which the medium is constructed, to have different elasticities. Such a structure represents a crystalline medium. Its vibrations would be propagated according to the laws of propagation of light in crystalline media. The wave-surface would be Fresnel's wave-surface, and it would exhibit conical refraction. If the cubes were twisted, the structure would be like that of quartz or other substances that rotate the plane of polarization of light. In order to represent the rotation of the plane of polarization of light by magnetism, it would be necessary to introduce some mechanism connecting this æther with matter. That this would be required is evident from the consideration that no amount of inherent rotation of these wheels would alter the plane of polarization of a vibration transmitted by them. Now, although I am not prepared to suggest any actual method of connection, it may be worth while pointing out two different ways in which it may work.

In the first place the matter may be in rotation when subject to magnetic force ; and it may be connected with the æther in such a way that the direction of its axis and rotation is altered by the vibrations passing in the æther. It would then react on the æther in such a way as to rotate the plane of polarization of the wave. In the second place the matter might only act as a link connecting æther elements rotating in rectangular directions in such a way that the rotation of one element altered the axis of rotation of a rectangular element. The reaction of this latter on the former would then rotate the plane of polarization of a wave propagated parallel to the latter. The first of these connections is that in accordance with Maxwell's theory as to the connection between the æther and matter that he introduces in his 'Electricity and Magnetism,' in order to explain the rotation of the plane of polarization of light by magnetism ; while the second is the one that I supposed in my paper "On the Electromagnetic Theory of the Reflection and Refraction of Light" (R. S. Trans. 1880).

I need hardly say, in conclusion, that I do not in the least intend to convey the impression that the actual structure of the æther is a bit like what I have described. What physicists ought to look for is such a mode of motion in space as will confer upon it the properties required in order that it may exhibit electromagnetic phenomena. Such a mode of motion would be a real explanation of these phenomena. I have only given a description of them.

I think, however, that it is worth while considering these models, because in them the disturbance which represents light is not the same as the vibrations of an elastic jelly, for what represents an electric displacement is a change of structure of an element, and not a displacement of the element ; and it seems almost certain that, notwithstanding the very high authority which seems to support the view that the æther is *like* an elastic jelly, nevertheless its vibrations are much more of the nature of alterations in structure than of displacements.